Technical Comments

Earliest Classic Result for the Turbulent Hydraulic Wake behind Body of Revolution

Raymond H. Cramer*
The Johns Hopkins University, Silver Spring, Md.

IN 1929 a thorough analysis of the hydraulic, turbulent-wake, mean-flow properties behind a body of revolution was presented by Swain in Ref. 1. At the first stage of approximation in her treatment, the results for the spreading of the half-width (centerline to outer edge) of the wake, b, and for the decay of the relative velocity deficit $(U_\infty - U)/U_\infty$ were determined to be.

$$b = (70/4\pi)^{1/5}(3c^2)^{1/5}(C_D F x)^{1/3} \quad (1)$$

and

$$\frac{U_\infty - U}{U_\infty} = \left(\frac{C_D F}{x^2}\right)^{1/3} \frac{\eta_0^3}{9}\left[1 - \left(\frac{r}{b}\right)^{3/2}\right]^2 \quad (2)$$

where

$$\eta_0^3 = 9A^2 = [70/(4\pi)]^{3/5}(3c^2)^{-2/5} \quad (3)$$

The notation used in these expressions is most easily explained by Fig. 1.

The quantities that need further comment are C_D, F, η_0, A, and c. In conventional notation C_D represents the drag coefficient for the body; i.e., drag = $C_D \cdot q \cdot F$, where F is the selected reference area and q is the dynamic head, more explicitly defined by $q = \rho U_\infty^2/2$, where ρ is the mass density of the constant-density medium in which the body is immersed. The constants A and η_0 are merely convenient intermediate evaluations, as mathematically defined in Eq. (3), that make the form of Eq. (2) neater, without masking the fact that both b and $(U_\infty - U)/U_\infty$ depend on the constant, c, which is to be empirically evaluated. It is worth clarifying the meaning of c by noting that it enters the analysis given in Ref. 1 in the form

$$l = c(C_D F x)^{1/3} \quad (4)$$

where l is the well-known mixing length of Prandtl. Inasmuch as it has become customary to introduce another constant, β, as the constant of proportionality between the mixing length, l, and the wake semiwidth, b, in the form $l = \beta b$, it can be readily determined that in terms of β the value of c is simply

$$c = (210)^{1/3}(4\pi)^{-1/3}\beta^{5/3} \quad (5)$$

As a matter of fact, the results for b and $(U - U)/U$, when converted to give explicit prominence to the constant β, are

$$b = (105)^{1/3}(2)^{-1}\beta^{2/3}(C_D d^2 x)^{1/3} \quad (1a)$$

and

$$(U_\infty - U)/U_\infty = (105)^{1/3}(54)^{-1}\beta^{-4/3}(C_D d^2)^{1/3}x^{-2/3}[1 - (r/b)^{3/2}]^2 \quad (2a)$$

where the reference area has been taken to be the circular cross-sectional area at the maximum width of the body, for which $F = \pi d^2/4$; i.e., the maximum body diameter is d.

These latter two expressions were rederived recently in Ref. 2, where it was stated that they are simpler than the solutions (1) and (2). It is clear, however, that there is no essential difference between the two sets of equations except insofar as there may be a preference for taking the empirical constant to be β rather than c. It needs to be emphasized that these results are what one obtains when terms only to order $x^{-5/3}$ are retained in the governing differential equation; the thorough analysis provided in Ref. 1 went on further to give the more complicated expressions which arise when terms to order $x^{-7/3}$ are retained in the controlling equation.

An evaluation of the empirical constant c (and thus of β) can be obtained by fitting the theoretical result to the experimentally determined data given by Chevray in Ref. 3. By careful reading of the numbers from both the sketched "asymptotic" curves (not the data points) provided by Figs. 17 and 18 in Ref. 3, it can be determined that to good accuracy $c = 0.0724$ (or $\beta = 0.118$). In consequence, the working curves, describing the wake developments which are derivable from Chevray's data, take the especially compact forms

$$b = 0.614(C_D F x)^{1/3} \quad (6)$$

$$(U_\infty - U)/U_\infty = 1.640(C_D F)^{1/3}x^{-2/3}[1 - (r/b)^{3/2}]^2 \quad (7)$$

It may be noted that for the prolate spheroid of 6 to 1 fineness ratio that was tested by Chevray the measured drag coefficient was found to be 0.060. Furthermore, the x-values are to be considered to start at a virtual origin behind the body, designated by x_0. Between the base of

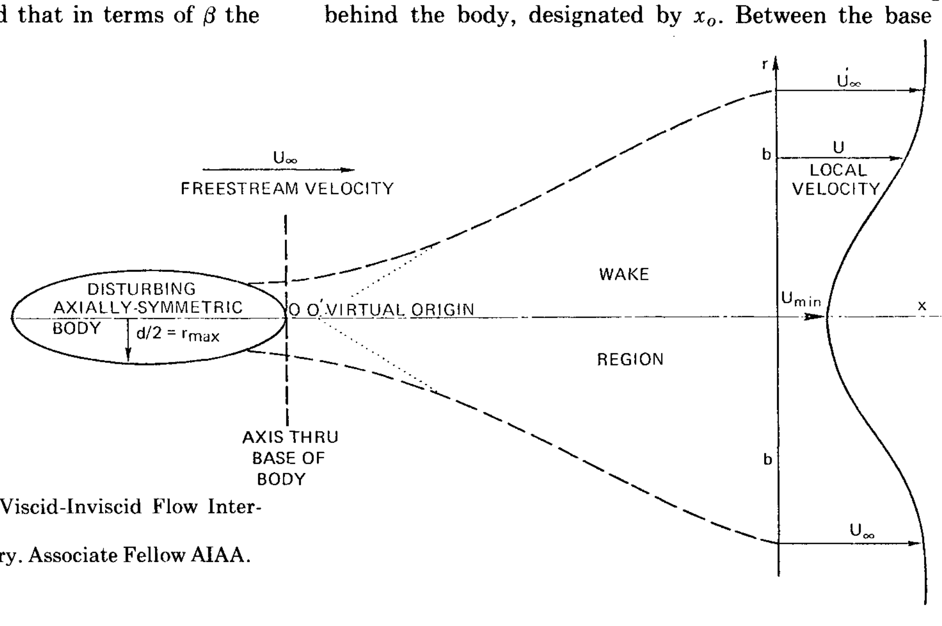

Fig. 1 Notation for axially symmetric wake problem.

Received November 27, 1972.
Index categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Hydrodynamics.

*Physicist, Applied Physics Laboratory. Associate Fellow AIAA.

the body and the virtual origin the observed wake converged, but at approximately 4 body-diameters downstream of the base the self-preservation or similarity region can be imagined to originate.

Unfortunately, it appears likely that the entire self-preservation and similarity concept does not represent closely what occurs in the wake in physical actuality. A recent paper by Ermshaus[4] gives very convincing proof that the supposed axially-symmetric wake-spreading behavior that would follow the $x^{1/3}$ law is not found to be confirmed by experiments, nor does the ratio remain constant that expresses the comparison between the maximum shear stress, represented by the quantity $\tau_m = -\rho(\overline{u'v'})_m$ (where u' and v' are the fluctuations of velocity in the x and r directions, respectively) to the square of the velocity defect at the middle of the wake. In the mixing-length theory used to derive the expressions given above there is implicit the hypothesis that $\tau_m/(U - U_\infty)_s{}^2$ must remain constant (where the s subscript denotes conditions at the centerline). The surprising result obtained by Ermshaus is that two-dimensional configurations (cylinders and narrow bands or beams) and axially symmetric configurations, as well, have to all intents and purposes the same exponent in the law describing the downstream spread of the wake. For all such configurations, to be precise, the width of the wake appears to grow approximately according to $x^{0.44} = x^{1/2.25}$, not the $x^{1/2}$ law predicted on basis of similarity theory for two-dimensional bodies, nor according to the $x^{1/3}$ law, predicted according to classic theory, for axially symmetric bodies. Consequently, even though Eqs. (6) and (7) appear to be well-substantiated by the faired data-plots given by Chevray, some caution should be exercised in trying to apply to quite different situations these particular results that have come from one kind of body shape and for which the asymptotic state apparently was not yet attained (if it ever would be).

References

[1]Swain, L. M., "On the Turbulent Wake behind a Body of Revolution," *Proceedings of the Royal Society of London*, Ser. A, Vol. 125, 1929, pp. 647–659.

[2]Chang, P. K. and Oh, Y. H., "Axially Symmetric Incompressible Turbulent Wake Downstream of a Single Body," *Journal of Hydronautics*, Vol. 2, No. 4, Oct. 1968, pp. 223-224.

[3]Chevray, R., "The Turbulent Wake of a Body of Revolution," *Transactions of the ASME, Ser. D: Journal of Basic Engineering*, Vol. 90, No. 21, June 1968, pp. 275–284.

[4]Ermshaus, R., "Typical Features of Turbulent Wake Flows," Translation CLB-3, T-650, 27 April 1972, The Johns Hopkins University, Applied Physics Lab., Silver Spring, Md.; also AD751970, from National Technical Information Service, Springfield, Va. 22151.

Reply by Authors to R. H. Cramer

P. K. Chang* and Y. H. Oh†

The Catholic University of America, Washington, D. C.

IN the case of axially-symmetric turbulent wake flow, the following equations for the velocity profile and the wake width were formulated simply from available information:

$$(U_\infty - U)/U_\infty = [x^2/(C_D d^2)]^{-1/3} f(\eta)$$

and

$$b = B(C_D dx)^{1/3} \tag{1}$$

where

$$\eta = y/b$$

Received December 28, 1972.

*Professor; also Consultant, U. S. Naval Ordnance Lab.

†Post Doctoral Fellow, NASA Langley Research Center, Hampton, Va.

These equations were then used to obtain the solution to the governing differential equations. The basis for formulating Eq. (1) is that information (Ref. 1, pp. 686, 691–2) applicable to two-dimensional wake flows, viz.,

$$(U_\infty - U)/U_\infty = [x/(C_D d)]^{-1/2} f(\eta)$$

$$b = B(C_D dx)^{1/2}$$

whereas for axially symmetric wake flow;

$$(U_\infty - U)/U_\infty \sim x^{-2/3}$$

and

$$b \sim x^{1/3}$$

Thus, by considering the dimensions of the velocity profile and the wake width, Eq. (1) was formulated.

L. M. Swain, in her pioneering 1929 paper,[2] elaborately worked out a similar form of Eq. (1) by using an order-of-magnitude analysis. At that time apparently no such information was available.

The boundary-layer equations, which are applicable for the wake (Ref. 1, p. 682) has been applied to obtain a solution. Because such a boundary-layer equation has been derived under the assumption of thicknesses that are small with respect to x, we obtain a solution which we consider more applicable to the far wake.

Swain considered, in addition, a second term involving $x^{-7/3}$ which is more significant in the near wake region as the body is approached.[2]

References

[1]Schlichting, H., *Boundary Layer Theory*, 6th ed., McGraw-Hill, New York, 1968, pp. 682, 686, 691-692.

[2]Swain, L. M., "On the Turbulent Wake behind a Body of Revolution," *Proceedings of the Royal Society (London)*, Vol. A125, 1929, pp. 648, 659.

Comment on "Added Mass of a Circular Cylinder in Contact with a Rigid Boundary"

Theodore R. Goodman*

Oceanics Inc., Plainview, N. Y.

THE added mass of a circle tangent to a ground plane that is determined in the issue of January 1972,[1] was first calculated by Taylor[2] over forty years ago. Taylor actually calculated the added masses of a family of lenses consisting of two circular arcs. The family passes from a flat plate through a single circle to a pair of tangent circles. This last limiting case may be regarded as a circle tangent to a ground plane. In addition to this family, Taylor also considered other two-dimensional sections having a line of symmetry. Among these are a section whose upper half is defined by a symmetric parabolic arc, one whose upper half is in the shape of an isosceles triangle and one whose upper half is defined by a pair of semicircles.

References

[1]Garrison, C. J., "Added Mass of a Circular Cylinder in Contact with a Rigid Boundary," *Journal of Hydronautics*, Vol. 6, No. 1, Jan. 1972, pp. 59–60.

[2]Taylor, J. L., "Some Hydrodynamical Inertia Coefficients," *Philosophical Magazine*, Ser. 7, Vol. 9, 1930, pp. 161–183.

Received April 14, 1972.

Index categories: Hydrodynamics; Marine Vessel Design (Including Loads).

*Vice President. Associate Fellow AIAA.